## Integrating Quantum Concepts into Cybersecurity

### Activity 1a: Tutorial

1. Express the Z basis in terms of the X basis. Note that all vectors are normalised in quantum computing and quantum information.

2. Show that the set $B_1 = \{|+\rangle, \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\}$ forms a spanning set for the qubit space H. Establish to what extent $B_1$ is an orthogonal, orthonormal and linearly independent set. Include definitions for each type of set.

3. For a qubit space with basis $\{|+\rangle, |-\rangle\}$ calculate the expansion of $\frac{1}{\sqrt{2}}(|+\rangle + e^{i\theta}|-\rangle)$ in terms of the basis $\{|0\rangle, |1\rangle\}$

4. Derive the density operator representations for vectors in the Z and X basis

5. Explain the difference between a *pure state* and a *mixed state*. Explain how one could distinguish between pure and mixed states. Hence establish whether $|\psi\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$ and $\rho = \cos^2\theta|0\rangle\langle 0| + \sin^2\theta|1\rangle\langle 1|$ describe pure or mixed states.

6. Verify that the eigenvectors of the Z Pauli gate is given by the Z basis and that the eigenvectors of the X Pauli gate is given by the X basis. What are the corresponding eigenvalues?

7. The representation of quantum states in terms of the Bloch Sphere is achieved via the relationship:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Given that $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, it may be shown that:

$$|\psi\rangle = \cos\theta/2|0\rangle + e^{i\phi}\sin\theta/2|1\rangle \text{ with } 0 \le \theta \le \pi$$

Using this second representation show that the X basis lies along the x axis in the Bloch sphere and the Z basis lies along the z axis in the Bloch sphere.

8. The gates that we have met are said to be examples of unitary gates. Explain what is meant by a unitary gate and hence show that the Hadamard, Pauli and Phase gates are all unitary gates.

9. Let $|\psi\rangle = \sin\frac{\theta}{4}|0\rangle + cos\frac{\theta}{4}|1\rangle$ and $|\varphi\rangle = \sin^2\frac{\theta}{4}|0\rangle\langle 0| + cos^2\frac{\theta}{4}|1\rangle\langle 1|$ denote states generated by two different statistical preparations. Establish the possible measurement outcomes of the observable $\sigma_X$ and calculate the likelihood of obtaining each outcome for each of the preparations. What will the expectation value $\langle\sigma_X\rangle$ and variance be for the state $|\psi\rangle = \sin\frac{\theta}{4}|0\rangle + cos\frac{\theta}{4}|1\rangle$?